

18th Benelux Mathematical Olympiad

Valkenswaard, 24–26 April 2026



The problems are not in order of estimated difficulty.

Problem 1. Let n be a positive integer and let $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n$ be positive real numbers such that

$$a_k(b_1 + \dots + b_k) \leq k \quad \text{and} \quad b_k(a_1 + \dots + a_k) \leq k$$

for $k = 1, 2, \dots, n$. Prove that $(a_1 + a_2 + \dots + a_n)(b_1 + b_2 + \dots + b_n) \leq n^2$.

Problem 2. Let a_0 and N be positive integers. Ben and Lux take turns playing the following game: At the start of the game, the number N is written on a board. At move $i \geq 1$ of the game, the player whose turn it is chooses a positive integer $a_i \leq a_{i-1}$, and replaces the number n on the board at that point with $n - a_i$. The first player to write down a number $n \leq 0$ loses the game. Ben makes the first move. For each $N \geq 2$, determine the smallest possible value of a_0 for which Ben can win the game irrespectively of how Lux plays.

Problem 3. Let $ABCD$ be a parallelogram such that $|AC| = |BC|$, and let A' be the reflection of A in B . Lines BD and $A'D$ meet the circumcircle of triangle ACD again at E and F , respectively. Lines AE and $A'C$ intersect at G . Show that lines AC , EF , BG are concurrent or pairwise parallel.

Problem 4. Find all pairs (a, b) of positive integers for which there exists a positive integer c such that $a^2 + b^2 + c^2$ divides $(a + b + c)^2$.

Language: English

Available time: 4 hours and 30 minutes
Each problem is worth 7 points