17th Benelux Mathematical Olympiad

25–27 April 2025 — Liège, Belgium

Problems

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Language: ***English***

Problems are ***not*** *ordered by estimated difficulty.*

Problem 1. Does there exist a function $f : \mathbb{R} \to \mathbb{R}$ such that

$$f\left(x^2 + f(y)\right) = f(x)^2 - y$$

for all $x, y \in \mathbb{R}$?

Problem 2. Let $N \geqslant 2$ be a natural number. At a mathematical olympiad training camp, the same N courses are organised every day. Each student takes exactly one of the N courses each day. At the end of the camp, every student has taken each course exactly once, and any two students took the same course on at least one day, but took different courses on at least one other day. What is, in terms of N, the largest possible number of students at the camp?

Problem 3. Let ABC be a triangle with incentre I and circumcircle Ω. Let D, E, F be the midpoints of the arcs $\overset{\frown}{BC}, \overset{\frown}{CA}, \overset{\frown}{AB}$ of Ω not containing A, B, C, respectively. Let D' be the point of Ω diametrically opposite to D. Show that I, D', and the midpoint M of $[EF]$ lie on a line.

Problem 4. Let $a_0, a_1, \ldots, a_{10}$ be integers such that, for each $i \in \{0, 1, \ldots, 2047\}$, there exists a subset $S \subseteq \{0, 1, \ldots, 10\}$ with

$$\sum_{j \in S} a_j \equiv i \pmod{2048}.$$

Show that for each $i \in \{0, 1, \ldots, 10\}$, there is exactly one $j \in \{0, 1, \ldots, 10\}$ such that a_j is divisible by 2^i but not by 2^{i+1}.

Note: $\sum_{j \in S} a_j$ *is the summation notation, for instance,* $\sum_{j \in \{2,5\}} a_j = a_2 + a_5$*, while, for the empty set* $\varnothing$*, one defines* $\sum_{j \in \varnothing} a_j = 0$.

Time allowed: 4 hours and 30 minutes
Each problem is worth 7 points