

# 14th Benelux Mathematical Olympiad

Leuven, April 29–May 1



*The problems are not ordered by estimated difficulty.*

**Problem 1.** Let  $n \geq 0$  be an integer and let  $a_0, a_1, \dots, a_n$  be real numbers. Show that there exists  $k \in \{0, 1, 2, \dots, n\}$  such that

$$a_0 + a_1x + a_2x^2 + \dots + a_nx^n \leq a_0 + a_1 + \dots + a_k$$

for all real numbers  $x \in [0, 1]$ .

**Problem 2.** Let  $n$  be a positive integer. There are  $n$  ants walking along a line at constant nonzero speeds. Different ants need not walk at the same speed or walk in the same direction. Whenever two or more ants collide, all the ants involved in this collision instantly change directions. (Different ants need not be moving in opposite directions when they collide, since a faster ant may catch up with a slower one that is moving in the same direction.) The ants keep walking indefinitely.

Assuming that the total number of collisions is finite, determine the largest possible number of collisions in terms of  $n$ .

**Problem 3.** Let  $ABC$  be a scalene acute triangle. Let  $B_1$  be the point on ray  $[AC$  such that  $|AB_1| = |BB_1|$ . Let  $C_1$  be the point on ray  $[AB$  such that  $|AC_1| = |CC_1|$ . Let  $B_2$  and  $C_2$  be the points on line  $BC$  such that  $|AB_2| = |CB_2|$  and  $|BC_2| = |AC_2|$ . Show that  $B_1, C_1, B_2, C_2$  are concyclic.

**Problem 4.** A subset  $A$  of the natural numbers  $\mathbb{N} = \{0, 1, 2, \dots\}$  is called *good* if every integer  $n > 0$  has at most one prime divisor  $p$  such that  $n - p \in A$ .

- (a) Show that the set  $S = \{0, 1, 4, 9, \dots\}$  of perfect squares is good.
  - (b) Find an infinite good set disjoint from  $S$ .
- (Two sets are *disjoint* if they have no common elements.)

*Language: English*

*Time available: 4 hours and 30 minutes  
Each problem is worth 7 points*