12th Benelux Mathematical Olympiad

Virtual, 2–3 May 2020

BxMO 2020

*The problems are not ordered by estimated difficulty.*

Problem 1. Find all positive integers d with the following property: there exists a polynomial P of degree d with integer coefficients such that $|P(m)| = 1$ for at least $d + 1$ different integers m.

Problem 2. Let N be a positive integer. A collection of $4N^2$ unit tiles with two segments drawn on them as shown is assembled into a $2N \times 2N$ board. Tiles can be rotated.
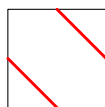
The segments on the tiles define paths on the board. Determine the least possible number and the largest possible number of such paths.

For example, there are 9 paths on the 4×4 board shown below.
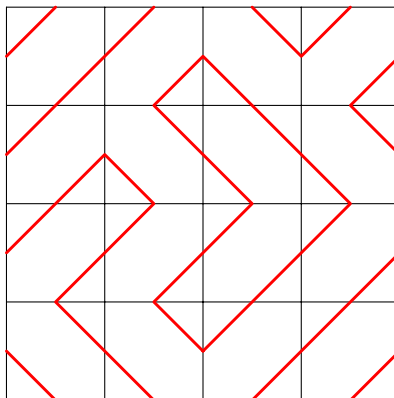
Problem 3. Let ABC be a triangle. The circle ω_A through A is tangent to line BC at B. The circle ω_C through C is tangent to line AB at B. Let ω_A and ω_C meet again at D. Let M be the midpoint of line segment $[BC]$, and let E be the intersection of lines MD and AC. Show that E lies on ω_A.

Problem 4. A divisor d of a positive integer n is said to be a *close* divisor of n if $\sqrt{n} < d < 2\sqrt{n}$. Does there exist a positive integer with exactly 2020 close divisors?

Language: English

Time available: 4 hours and 30 minutes
Each problem is worth 7 points