11th Benelux Mathematical Olympiad

Valkenswaard, 26–28 April 2019

The problems are not ordered by estimated difficulty.

Problem 1.

a) Let a, b, c, d be real numbers with $0 \leqslant a, b, c, d \leqslant 1$. Prove that

$$ab(a-b) + bc(b-c) + cd(c-d) + da(d-a) \leqslant \frac{8}{27}.$$

b) Find all quadruples (a, b, c, d) of real numbers with $0 \leqslant a, b, c, d \leqslant 1$ for which equality holds in the above inequality.

Problem 2. Pawns and rooks are placed on a 2019×2019 chessboard, with at most one piece on each of the 2019^2 squares. A rook *can see* another rook if they are in the same row or column and all squares between them are empty. What is the maximal number p for which p pawns and $p + 2019$ rooks can be placed on the chessboard in such a way that no two rooks can see each other?

Problem 3. Two circles Γ_1 and Γ_2 intersect at points A and Z (with $A \neq Z$). Let B be the centre of Γ_1 and let C be the centre of Γ_2. The exterior angle bisector of $\angle BAC$ intersects Γ_1 again at X and Γ_2 again at Y. Prove that the interior angle bisector of $\angle BZC$ passes through the circumcentre of $\triangle XYZ$.

For points P, Q, R that lie on a line ℓ in that order, and a point S not on ℓ, the interior angle bisector *of $\angle PQS$ is the line that divides $\angle PQS$ into two equal angles, while the* exterior angle bisector *of $\angle PQS$ is the line that divides $\angle RQS$ into two equal angles.*

Problem 4. An integer $m > 1$ is *rich* if for any positive integer n, there exist positive integers x, y, z such that $n = mx^2 - y^2 - z^2$. An integer $m > 1$ is *poor* if it is not rich.

a) Find a poor integer.

b) Find a rich integer.

Language: English

Time available: 4 hours and 30 minutes
Each problem is worth 7 points