Problem 1. Find the greatest positive integer $N$ with the following property: there exist integers $x_1, \ldots, x_N$ such that $x_i^2 - x_ix_j$ is not divisible by 1111 for any $i \neq j$.

Problem 2. Let $n$ be a positive integer. Suppose that its positive divisors can be partitioned into pairs (i.e. can be split in groups of two) in such a way that the sum of each pair is a prime number. Prove that these prime numbers are distinct and that none of these are a divisor of $n$.

Problem 3. Find all functions $f: \mathbb{R} \to \mathbb{Z}$ such that

$$
\left( f(f(y) - x) \right)^2 + f(x)^2 + f(y)^2 = f(y) \cdot \left( 1 + 2f(f(y)) \right)
$$

for all $x, y \in \mathbb{R}$.

Problem 4. A circle $\omega$ passes through the two vertices $B$ and $C$ of a triangle $ABC$. Furthermore, $\omega$ intersects segment $AC$ in $D \neq C$ and segment $AB$ in $E \neq B$. On the ray from $B$ through $D$ lies a point $K$ such that $|BK| = |AC|$, and on the ray from $C$ through $E$ lies a point $L$ such that $|CL| = |AB|$. Show that the circumcentre $O$ of triangle $AKL$ lies on $\omega$.

Language: English

Time available: 4.5 hours

Each problem is worth 7 points