4th Benelux Mathematical Olympiad

20-22 April 2012 – Namur – Belgium

Problems

Language: **English**

Problem 1. A sequence a_1, a_2, $\ldots$, a_n, $\ldots$ of natural numbers is defined by the rule

$$a_{n+1} = a_n + b_n \quad (n = 1, 2, \ldots)$$

where b_n is the last digit of a_n. Prove that such a sequence contains infinitely many powers of 2 if and only if a_1 is not divisible by 5.

Problem 2. Find all quadruples (a, b, c, d) of positive real numbers such that $abcd = 1$, $a^{2012} + 2012b = 2012c + d^{2012}$ and $2012a + b^{2012} = c^{2012} + 2012d$.

Problem 3. In triangle ABC the midpoint of BC is called M. Let P be a variable interior point of the triangle such that $\angle CPM = \angle PAB$. Let Γ be the circumcircle of triangle ABP. The line MP intersects Γ a second time in Q. Define R as the reflection of P in the tangent to Γ in B. Prove that the length $|QR|$ is independent of the position of P inside the triangle.

Problem 4. Yesterday, $n \geq 4$ people sat around a round table. Each participant remembers only who his two neighbours were, but not which one sat on his left and which one sat on his right. Today, you would like the same people to sit around the same round table so that each participant has the same two neighbours as yesterday (it is possible that yesterday's left-hand side neighbour is today's right-hand side neighbour). You are allowed to query some of the participants: if anyone is asked, he will answer by pointing at his two neighbours from yesterday.

(a) Determine the minimal number $f(n)$ of participants you have to query in order to be certain to succeed, if later questions must not depend on the outcome of the previous questions. That is, you have to choose in advance the list of people you are going to query, before effectively asking any question.

(b) Determine the minimal number $g(n)$ of participants you have to query in order to be certain to succeed, if later questions may depend on the outcome of previous questions. That is, you can wait until you get the first answer to choose whom to ask the second question, and so on.

Time allowed: 4 hours and 30 minutes
Each problem is worth 7 points