Language **English**

2nd Benelux Mathematical Olympiad
Amsterdam, 23–25 April 2010

Problems

Problem 1. A finite set of integers is called *bad* if its elements add up to 2010. A finite set of integers is a *Benelux-set* if none of its subsets is bad. Determine the smallest integer n such that the set $\{502, 503, 504, \dots, 2009\}$ can be partitioned into n Benelux-sets.
(A partition of a set S into n subsets is a collection of n pairwise disjoint subsets of S, the union of which equals S.)

Problem 2. Find all polynomials $p(x)$ with real coefficients such that

$$p(a+b-2c)+p(b+c-2a)+p(c+a-2b)=3p(a-b)+3p(b-c)+3p(c-a)$$

for all $a, b, c \in \mathbb{R}$.

Problem 3. On a line l there are three different points A, B and P in that order. Let a be the line through A perpendicular to l, and let b be the line through B perpendicular to l. A line through P, not coinciding with l, intersects a in Q and b in R. The line through A perpendicular to BQ intersects BQ in L and BR in T. The line through B perpendicular to AR intersects AR in K and AQ in S.

(a) Prove that P, T, S are collinear.

(b) Prove that P, K, L are collinear.

Problem 4. Find all quadruples (a, b, p, n) of positive integers, such that p is a prime and

$$a^3 + b^3 = p^n.$$

Language: English

Time allowed: 4 hours and 30 minutes
Each problem is worth 7 points